



# **Numerical Methods for Underwater Structural Acoustics**

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## **A. Scientific Research Goals**

This report represents the status of the project referenced above as of June 30, 1995, the conclusion of the term of this grant.

The project centered on a problem in structural acoustics: to determine the reflection properties of an underwater object interacting with sonar signals. To generate representative problems, we have been using the SARA package produced by H. Alik and D. Moore of Bolt Beranek and Newman, New London, CT. This code uses a finite/infinite (quadratic) element formulation in the frequency domain. The resulting linear system, which is complex symmetric, is currently solved with heavy dependence on secondary storage (i.e., disk) by Gaussian elimination using a frontal approach, with no pivoting for stability. This method has given adequate numerical results, even though it might be expected to fail due to resonances in the system. It is also likely to be too costly to handle large problems, especially for three dimensional models. Our goal has been to investigate two alternative approaches that we expect will alleviate these difficulties: the use of iterative methods and the use of parallel sparse solvers.

The exploratory results in this report will serve as the basis for work in the second phase of the project.

## **B. Significant Results**

### **B.1. Use of Iterative Methods: Elman, O'Leary, Stewart**

This research was motivated by previous experience suggesting that although direct methods are highly efficient for two-dimensional problems, a well-chosen iterative method can provide substantial computational improvement over direct solution on three-dimensional problems. This must be tempered by the fact that little is known about the behavior of iterative methods on systems such as these, even for one-dimensional and two-dimensional problems.

Our investigations are focused on the potential of various algorithms as alternatives to direct solution of the linear system. We experimented with two iterative methods:

1. The QMR algorithm of Freund and Nachtigal.

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2. The LSQR algorithm of Paige and Saunders.

The original formulation of the problem leads to a *graded* matrix, in which the first rows are much larger in magnitude than the last. Therefore, we allowed two different scalings:

1. Working with the original matrix  $A$ .
2. Working with the matrix  $\hat{A} \equiv D^*AD$ , where  $D$  is a diagonal matrix chosen to make the diagonal elements of  $\hat{A}$  equal to 1.

Several preconditioners  $M$  were considered.

1. No preconditioning:  $M \equiv I$ .
2. Preconditioning by the diagonal blocks corresponding to the structure, the water, and the boundary between the two.
3. Preconditioning by a banded matrix matching some of the elements of the diagonal blocks of the original matrix
4. The Huckle-Grote preconditioner, which constructs a sparse approximate inverse of  $A$ .

Preconditioning can be performed on either the left (corresponding to a rescaling of the equations) or on the right (corresponding to a change of variables). Right preconditioning has been moderately more successful in our tests.

The bulk of our experiments were performed with two BBN test problems: one with 1882 degrees of freedom corresponding to a 100 Hz plane wave incident to a flexible sphere of radius 1 meter and thickness 1 cm; and the other with 3649 degrees of freedom derived from a point load on a submerged flexible cylinder with spherical endcaps, where the cylinder radius had 1 meter, length of cylinder 3 meters, and wall thickness 1 cm.

We have had great success in using the diagonal block preconditioner. The residual norm is reduced by a factor of  $10^{-4}$  in fewer than 10 QMR iterations, while the relative error in the solution drops by a comparable factor. This algorithm is the iterative equivalent of *domain decomposition*, in which linear systems approximating partial differential equations are solved in each of two independent subdomains, one corresponding to the structure and one corresponding to the water, and the results are then "glued" together by the iterative method. This success in decoupling the structure from the water opens the possibility we can use different, specially tailored iterative methods on each part.

We have also studied the algebraic structure of the matrix to investigate whether a preconditioner that involved only the real part of the matrix (ignoring the imaginary

components) or a banded portion of the matrix would be successful. The preconditioner corresponding to the real part of the matrix is not adequate, but a banded preconditioner can be successful if done carefully. For example, taking 90% of each diagonal block does not lead to a practical algorithm, but convergence can be obtained by using 100% of both the block for the structure and the coupling block between elements in the water and those in the structure, together with 70% of the block for the water elements.

Using incomplete factorization as a preconditioner did not prove to be economical. We also showed that there is no scaling of the matrix that makes all nonzero elements of comparable size.

Domain decomposition on the water blocks produced slow convergence for nonoverlapping blocks and no convergence with overlapping blocks. An SP2 implementation was completed.

In other work on parallel computers, we have performed some tests of iterative methods on the Connection Machine 5, for the Helmholtz equation with radiation boundary conditions. This problem is similar in form to the block subproblems on the individual subdomains. Results for two-dimensional problems indicate that iteration counts for problems on an  $n \times n$  grid are proportional to  $n$ , somewhat better than expected for an indefinite problem.

We also developed methods to exploit the parametric nature of the problem. Experiments with the 2-dimensional Helmholtz equation showed that a preconditioner based on a fixed frequency could be effectively used for a whole range of nearby frequencies.

## **B.2. Parallel Sparse Factorization: Saltz**

This work concerned the parallel solution of the finite element equations by a parallel sparse direct solver. The single processor equivalent is the SARA program, which is well implemented and probably not subject to much improvement. Consequently, any speedup in the direct solution of the system must come from the ability to use more than one processor.

We have developed (in collaboration with the IBM Thomas J. Watson Research Center) a sparse Cholesky code that exploits the dense structures within the sparse matrix to obtain good single processor performance. We have extended it to a variety of distributed memory architectures. Our approach partitions the sparse matrix into regular chunks of nonzeros, which we call a *block*. Since the blocks have very regular internal nonzero structure, a significant part of the numerical computations can be carried out using efficient dense matrix kernels. We can therefore make use of Level-3 BLAS subroutines, which are highly optimized for modern architectures, to carry out the numerical computations.

For acoustics problems, we have produced a complex version of this solver and tested the code with the two problems generated by SARA discussed in Section B.1 as well as

a variant of the second problem, on both an Intel Paragon and a Cray T3D. A summary of the megaflop rates achieved in these tests (using double precision arithmetic) is given below.

	Matrix	1	2	4	8	16	32	64	128
Paragon	PROB1	14.16	21.4	34.3	60.1	86.2	124.4	189.2	294.3
	PROB2	13.9	22.4	37.5	62.7	97.2	131.5	194.2	303.4
	PROB3	14.2	21.9	36.2	63.3	95.3	132.7	190.1	301.6
T3D	PROB1	17.3	31.4	56.1	93.7	158.4	241.6	360.7	604.1
	PROB2	16.4	30.7	55.8	97.4	169.2	254.0	372.1	611.9
	PROB3	17.1	29.1	53.4	91.3	148.5	235.4	352.3	598.1